

# Design of a Cyclic Multiverse

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Recently, it has been noticed that the amplification of the amplitude of curvature perturbation cycle by cycle can lead to a cyclic multiverse scenario, in which the number of universes increases cycle by cycle. However, this amplification will also inevitably induce either the ultimate end of corresponding cycle, or the resulting spectrum of perturbations inside corresponding universe is not scale invariant, which baffles the existence of observable universes. In this paper, we propose a design of a cyclic multiverse, in which the observable universe can emerges naturally. The significance of a long period of dark energy before the turnaround of each cycle for this implementing is shown.

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Recently, a cosmological cyclic scenario, in which the universe experiences the periodic sequence of contractions and expansions [1], has been reawaked [2], and brought the distinct insights into the origin of observable universe. There has been lots of studies for cyclic or oscillating universe [3],[4],[5],[6],[7],[8],[9],[10],[11],[12], also [13] for a review. However, the global configuration of cyclic universe is actually more complex than imagined. In cyclic universe the amplitude of the curvature perturbation on super horizon scale is generally increasing during the contraction of each cycle, while is nearly constant during the expansion of each cycle. Thus the net result is that the amplitude of perturbation is amplified, which occurs cycle by cycle,

Recently, it has been argued that the amplification of the amplitude of curvature perturbation cycle by cycle can lead to a cyclic multiverse scenario [14]<sup>1</sup>, in which the universe proliferates, i.e. the number of universes increases cycle by cycle. There this was illustrated by including a contracting phase with  $w \simeq 0$  in each cycle of a cycle universe. In principle, since shortly after the end of each cycle the curvature perturbation on super horizon scale can be amplified to order one, the universe will be inevitably separated into lots of parts independent of one another after each cycle, each of which actually corresponds to a new universe and independently evolves up to succedent cycle, and then proliferates again<sup>2</sup>. This result in some sense incorporates the second law of thermodynamics in such a fashion that the increase of total entropy in consecutive cycles is explained as or replaced

with the increase of the number of new universes, which has been discussed in [14] in detailed.

The cyclic multiverse scenario might be significant for understanding the origin of observable universe. However, generally the amplitude of perturbation modes that enter into the horizon during the expansion in previous cycle and then leave it during the contraction in current cycle is generally larger than that induced by the quantum fluctuation of background field in current cycle. This will inevitably lead to either the ultimate end of corresponding cycle, or the resulting spectrum of perturbations inside each universe is not scale invariant even if they leave the horizon during the contraction with  $w \simeq 0$ , which baffles possible existence of observable universes in this scenario. We, in this paper, will discuss this problem in detail and then design a possible solution.

We begin with an illustration of cyclic universe model and the review of the evolution of perturbation in a cyclic universe with the contraction with  $w \simeq 0$ . There have been lots of studies of bounce cosmology [13]. Recently, the nonsingular bounce has been implemented in non-local higher derivative theories of gravity [19], which is ghostfree, while here that provided by us is only a simple example serving the purpose of illustration. We introduce a normal field  $\varphi$  with its potential  $M^2\varphi^2 - \Lambda_*$ , and a field  $\chi$  with negative  $\chi^2$ , which plays a crucial role in giving a nonsingular bounce [20].  $\Lambda_*$  is a small positive constant. Thus the minimum of potential is negative, which is responsible for the turnaround of cyclic universe. We show such an illustration of cyclic universe in Fig.1, in which  $M = 0.9$ ,  $\Lambda_* = 10^{-10}$ , the initial value  $10^4\varphi_0 = 5M$  and  $10^4\chi_0 = 3M$  are used and  $M_P = 1$  is set. It seems that this model suffers from the quantum instabilities and inconsistencies [19]. However, it can be expected that this effective description is only an approximation of a fundamental theory with the UV completion below certain physical cutoff, while the appearance of ghost terms or the quantum instabilities is only an artefact of this approximation. In string theory, a slowly decaying D3 brane can be depicted by an open string tachyon mode, which is described within a open string field theory. This will bring a nonlocal polynomial interaction. In this case, the behavior of open string tachyon can be effectively simulated by a ghost field, e.g.[21],[22],

<sup>1</sup> Here, the cyclic multiverse means that there are many independent universes in each cycle or each spacelike slice. While in usual cyclic universe there is only single universe in each cycle or each spacelike slice, which can be regarded as a multiverse only when we count it along the time sequence.

<sup>2</sup> In [15], an eternal expanding recurrent universe, in which  $h$  oscillates periodically while  $a$  expands all along, has been proposed phenomenologically, also latest [16], which can be implemented by appealing to a phantom component with  $w < -1$ . In this scenario, we live in one period of cycling, while the present acceleration with  $w < -1$  is just a start of the phantom inflation [17],[18] for the next period of cycling. Similarly, a multiverse scenario might be obtained here, which will be explored.

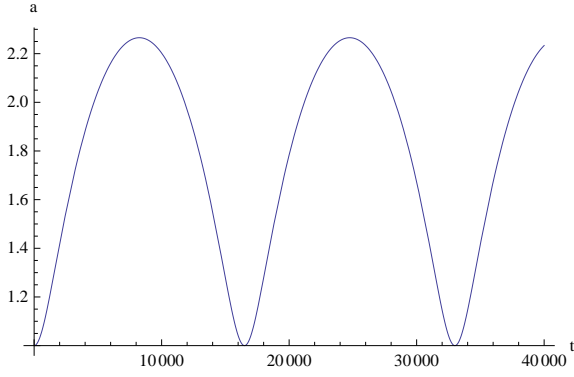


FIG. 1: A simple model of cyclic universe

as used here. However, the string theory is expected to be UV complete.

We will regard the beginning of the contracting phase as the beginning of a cycle, in each cycle the universe will orderly experience the contraction, bounce, and expansion, and then arrive at the turnaround, which signals the end of a cycle. The motive equation for perturbation in the momentum space is

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0, \quad (1)$$

where  $u_k$  is related to the curvature perturbation  $\zeta$  by  $u_k \equiv z\zeta_k$  [23],[24], and the prime denotes the derivative with respect to the conformal time  $\eta$ , and  $z = \varphi'/h$ , where  $h$  is the Hubble parameter and  $\varphi$  is the background field. When  $w \simeq 0$ ,  $z \sim \eta^2$ . Thus we have  $\frac{z''}{z} \sim \frac{2}{\eta^2}$ , which is actually the same as that in inflation, in which  $a \sim \frac{1}{\eta}$  leads  $z \sim \frac{1}{\eta}$  and thus  $\frac{z''}{z} \sim \frac{2}{\eta^2}$ . This gives the spectrum  $n_s \simeq 1$  is scale invariant, as has been shown in [25, 26], also see [27],[28],[29] and earlier [30] for tensor perturbation. Thus the amplitude of curvature perturbation is given by

$$\mathcal{P}_\zeta^{1/2} \simeq k^{3/2} \left| \frac{u_k}{z} \right| \simeq \frac{h_e}{m_p}, \quad (2)$$

where we neglected the factor with order one, and  $h_e$  is determined by the energy scale  $\rho_e$  at the end time of contracting phase,  $h_e \simeq \frac{\sqrt{\rho_e}}{m_p}$ . This amplitude of perturbation spectrum is actually determined by the increasing mode of metric perturbation  $\Phi$  during the contraction, which enters into the constant mode of  $\zeta$  or  $\Phi$  after the bounce by  $k^2$  order of  $\zeta$ . The amplitude of curvature perturbation during the contraction is increased, up to the end of contracting phase in corresponding cycle, while during the expansion it becomes constant on super horizon scale. Thus for a cycle the net result is the amplitude of curvature perturbation on super horizon scale is amplified, which is inevitable here.

We then consider the evolution of perturbation, which is generated in previous cycle, entering into current cy-

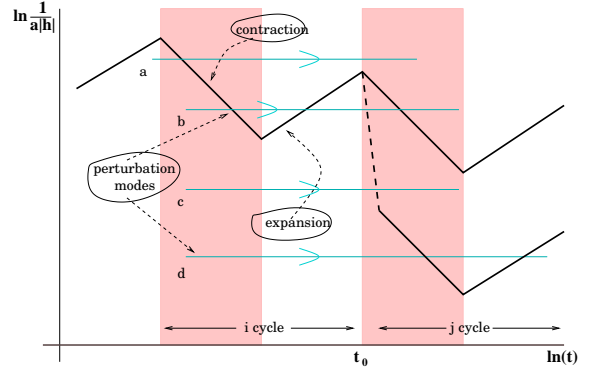


FIG. 2: In a cyclic universe, the sketch of  $\ln(\frac{1}{a|h|})$  vs.  $\ln(t)$ . The adjacent cycles have been signed as  $i$  and  $j$  cycles, respectively. The blue lines denote the evolutions of perturbation modes, and the red regions denote the contracting phases in each cycle. The dashed line denotes the period of dark energy domination, and the following solid line denotes the evolution after it.  $t_0$  is the turnaround epoch. In principle, at the bounce and turnaround points,  $h = 0$ , thus there is a divergence for  $\frac{1}{a|h|}$ , which is not plotted here and also Fig.3 and 4. However, the discussions are not affected by this neglect.

cle. We regard adjacent cycles as  $i$  and  $j$  cycles for convenience, respectively. The solution of metric perturbation  $\Phi$  is  $\Phi \simeq \Phi_C + \Phi_S h/a$ . When  $w \simeq 0$ , during the contraction the constant mode  $\Phi_C \sim \sqrt{k^3}$  and the increased mode  $\Phi_S \sim 1/\sqrt{k^7}$ . The solutions of  $\zeta$  is generally  $\zeta \simeq \Phi_C + k^2 \Phi_S f(\eta)$  [26], where

$$f(\eta) = \int \frac{d\eta}{z^2} \sim \frac{1}{\eta^3} \sim h \quad (3)$$

for  $w \simeq 0$ , since  $z \sim \eta^2$  and  $h \sim 1/\eta^3$ . There is not bounce, but the contraction phase after  $t_0$ . Thus  $\zeta$  is dominated by its increasing mode  $\Phi_S$ , i.e.  $\zeta \simeq k^2 \Phi_S f(\eta) \sim k^{-3/2} h$ . Thus  $\mathcal{P}_\zeta^{1/2} \simeq k^{3/2} |\zeta| \sim h$  is scale invariant. In  $j$  cycle, since at the beginning time  $t_0$ ,  $h = h_0$ , see Fig.2, and the initial value of perturbation is  $\mathcal{P}_{\zeta(i)}^{1/2}$ , the amplitude of perturbation mode all along on super horizon scale, which is generated during the contraction in  $i$  cycle and can not reenter into the horizon during the expansion of  $i$  cycle, see 'a' mode in Fig.1, is given by

$$\mathcal{P}_{\zeta(ji)}^{1/2} = \mathcal{P}_{\zeta(i)}^{1/2} \left( \frac{h_j}{h_0} \right) = \mathcal{P}_{\zeta(i)}^{1/2} e^{3\mathcal{N}_j}, \quad (4)$$

where the subscript  $i$  and  $j$  denote the quantities in corresponding cycles, respectively, and  $\mathcal{N}_j = \frac{1}{3} \ln(\frac{h_j}{h_0})$  is the e-folding number that the perturbation mode  $k$  lasts after the beginning of contracting phase in  $j$  cycle.

The amplitude of perturbation responsible for the large scale structure of observable universe is  $\mathcal{P}_\zeta^{1/2} \sim 10^{-5}$ . Thus if  $\mathcal{P}_{\zeta(i)}^{1/2} \sim 10^{-5}$  is required in  $i$  cycle, we can see when  $\mathcal{N}_j \simeq \frac{5}{3} \ln 10 \sim 3$ ,  $\mathcal{P}_{\zeta(ji)}^{1/2} \sim 1$  in  $j$  cycle, where

it should be noticed that when  $\mathcal{P}_{\zeta(ji)}^{1/2}$  approaches 1, the enhancement of nonlinear effect will make the required  $\mathcal{N}_j$  less. Thus this actually means that nearly at the beginning time of  $j$  cycle, the curvature perturbation on super horizon scale will have the amplitude be in order one. This will lead to the density perturbation

$$\frac{\delta\rho}{\rho} \sim \mathcal{P}_{\zeta(ji)}^{1/2} \sim 1 \quad (5)$$

on corresponding super horizon scale. In this case, it is obviously impossible that the different regions of global universe will evolve synchronously, even if they are synchronous in  $i$  cycle. This means that the global universe at the beginning time of  $j$  cycle will be inevitably separated into many different parts, each of which actually corresponds to a new universe and will evolve independently of one another, up to succedent cycle<sup>3</sup>. Therefore, it is evident that the global universe will proliferate after each cycle. In this sense, a cyclic multiverse actually comes into being.

However, inside each new universe, generally the amplitude of primordial perturbation, see ‘b’ mode in Fig.1, is contributed by not only that of perturbations induced by the quantum fluctuation of background field in  $j$  cycle, but also that of perturbations that enter into the horizon during the expansion of  $i$  cycle and then leave it during the contraction of  $j$  cycle<sup>4</sup>. In general, the latter amplitude can be larger, and dependent of the expansion behavior and the matter contents in  $i$  cycle, its spectrum is also not scale invariant. This can be seen as follows.

When  $k_*\eta \simeq 1$  during  $i$  cycle,  $k_*$  mode just enters into the horizon. Hereafter, its evolution obeys Eq.(1), but the term  $\frac{z''}{z}$  is negligible, since  $k \gg ah$ . In this case,  $u_k$  is approximately constant, up to the time when the corresponding mode leaves the horizon<sup>5</sup>, which occurs during the contraction in  $j$  cycle. Thus we have  $|u_{i*}| = |u_{j*'}|$ , where  $*$  is the time when the corresponding mode leaves the horizon in  $j$  cycle, which implies  $\mathcal{P}_{j*'}^{1/2} = (\frac{a_{i*}}{a_{j*'}})\mathcal{P}_{i*}^{1/2}$ . However, after this mode leaves the horizon in  $j$  cycle, its amplitude will increase like Eq.(4). Thus

$$\begin{aligned} \mathcal{P}_{je}^{1/2} &= \left(\frac{h_{je}}{h_{j*'}}\right) \mathcal{P}_{j*'}^{1/2} \\ &= \left(\frac{k_{je}}{k_{j*'}}\right)^3 \left(\frac{a_{i*}}{a_{j*'}}\right) \mathcal{P}_{i*}^{1/2} \end{aligned} \quad (6)$$

is obtained, where  $h \sim (ah)^3 \sim k^3$  for  $w \simeq 0$  has been applied, and the subscript ‘ $je$ ’ denotes the end time of the contracting phase in  $j$  cycle. We can see that if  $\mathcal{P}_{i*}^{1/2} \sim 10^{-5}$ ,  $\mathcal{P}_{je}^{1/2}$  will be larger than that induced by the quantum fluctuation of background field in  $j$  cycle since  $k_{j*'} \ll k_{je}$  and  $a_{i*} \simeq a_{j*}$ , and also the spectrum is quite red since  $\mathcal{P}_{je}^{1/2} \sim k_{j*'}^{-3}$ . This result implies that there can hardly an observable universe after the bounce of  $j$  cycle. In addition, it can be noticed that the amplitudes of these modes may be also amplified up to order one, i.e. after the contraction lasts some times,  $\mathcal{P}_j^{1/2} \sim 1$ . In this case, the universe will be possibly split into smaller and smaller fragments, which will renders the cycle ultimately end [14], see also [32]. Thus in this sense the feasibility of such a cyclic multiverse scenario is questionable.

The perturbation modes that enter into the horizon during the expansion of  $i$  cycle and then leave it during the contraction of  $j$  cycle are baneful. Their amplitudes can hardly be suppressed by certain mechanism. Thus a reasonable solution is to push these baneful modes to larger scale, i.e. outside of observable universe in corresponding cycle. This can be implemented by introducing a period of accelerated expansion in each cycle. This period can be set before the turnaround of each cycle. In this case, there will be a period of dark energy domination in corresponding cycle, hereafter the universe collapses and the next cycle begins. The perturbation modes that enter into the horizon during the expansion of  $i$  cycle will possibly leave it during the dark energy domination of  $i$  cycle, but not during the contraction of  $j$  cycle. Thus the amplitude of these modes will not increase, up to the turnaround. In this case,  $\mathcal{P}_{je}^{1/2} = (\frac{h_{je}}{h_0})(\frac{a_{i*}}{a_0})(\frac{a_0}{a_{j*'}})\mathcal{P}_{j*'}^{1/2}$ , where  $t_0$  in Fig.2 is the time when the period of dark energy domination begins, thus Eq.(6) is changed as

$$\mathcal{P}_{je}^{1/2} = \left(\frac{k_{je}}{k_{j0}}\right)^3 \left(\frac{k_0}{k_{i*}}\right)^3 \mathcal{P}_{i*}^{1/2}, \quad (7)$$

where  $a \sim (ah)^{\frac{n}{n-1}} \sim k^{\frac{n}{n-1}}$  has been applied, which gives that for  $n = \frac{2}{3}$ ,  $a \sim \frac{1}{k^2}$  and for  $n \gg 1$ ,  $a \sim k$ .  $k_{je}/k_{j0}$  denotes the e-folding number of primordial perturbation contributed by the contraction in  $j$  cycle, i.e.  $\mathcal{N}_j = \ln(\frac{k_{je}}{k_{j0}})$ . The larger it is, the larger the amplitude of perturbation is amplified during the contraction is. This point is essentially the same as that obtained in Eq.(4). While  $k_0/k_{i*}$  is the ratio of the wavenumber of the mode that corresponds to the beginning time of dark energy domination to that of the given mode, which actually corresponds to the e-folding number that the dark energy phase lasts before the given mode leaves the horizon during the dark energy domination of  $i$  cycle, and is an exponentially suppressed factor to the amplitude  $\mathcal{P}_{je}^{1/2}$ . This result means that the period of dark energy domination not only helps to the continuance of cycling but also the emergence of observable universe.

This design can be explained detailed in Fig.2. ‘a’

<sup>3</sup> It has been argued that if a region with super horizon scale has the density perturbation on corresponding scale larger than 1, such a region will correspond to a separated close universe [31]. In principle, the initial conditions of each local universe can be obtained by rescaling the background of parent universe.

<sup>4</sup> We thank R. Brandenberger for talking this point to us.

<sup>5</sup> We neglected the effect of the transfer function on the perturbation spectrum after the corresponding perturbation mode enters into the horizon, which is dependent of the matter content in corresponding cycle. However, the behaviors of spectrum discussed here are not altered qualitatively

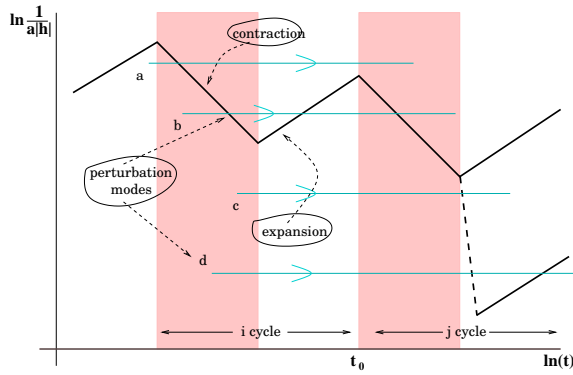


FIG. 3: In a cyclic universe, the sketch of  $\ln(\frac{1}{a|h|})$  vs.  $\ln(t)$  for the case that there is a period of inflation before bounce in  $j$  cycle. The blue lines denote the evolutions of perturbation modes, and the red regions denote the contracting phases in each cycle. The dashed line denotes the inflation, and the following solid line denotes the evolution after inflation.

mode denotes the modes that leave the horizon in  $i$  cycle but can not enter into the horizon during the expansion of  $i$  cycle. These modes will destined to stay on super horizon scale all along up to  $j$  cycle. Their amplitudes will be inevitably amplified to order one around the beginning time of  $j$  cycle. This leads to that the universe is separated into lots of independent new universes, each of which will evolve independently up to the succedent cycle. ‘b’ mode denotes the modes that enters into the horizon during the expansion of  $i$  cycle and then leaves it during the contraction of  $j$  cycle. These modes generally have amplitudes larger than that induced only by the fluctuation of background field in  $j$  cycle, which thus is baneful for the existence of observable universe in  $j$  cycle. The dashed line denotes the period of dark energy domination, which can push those baneful modes, such as ‘b’ mode, outside of observable universe. In this case, ‘d’ mode will provide the primordial perturbation of observable universe, which is induced only by the quantum fluctuation of background field in  $j$  cycle, and its amplitude is given by (2) and thus can be suitable for observations <sup>6</sup>.

<sup>6</sup> The inflation might occur after the bounce in some cycles, see Fig.3, since the energy scale of bounce is generally high, it can be expected that after the bounce the field might land on a higher plain of effective potential. In this case, ‘b’ mode will be pushed outside of observable universe by the inflation in  $j$  cycle, and the primordial perturbations suitable for observable universe, such as ‘c’ and ‘d’ modes, can emerge during inflation, which are induced by the quantum fluctuation of inflaton field. The inflation after bounce has been originally studied in Refs. [33, 34], in which the imprint of bounce on cosmic microwave background has been pointed out, see also for latest studies [35],[36]. Here for cyclic multiverse, the occurrence of inflation has similar effect as that of a period of dark energy domination. However, both can be different in the theoretical model building and observational

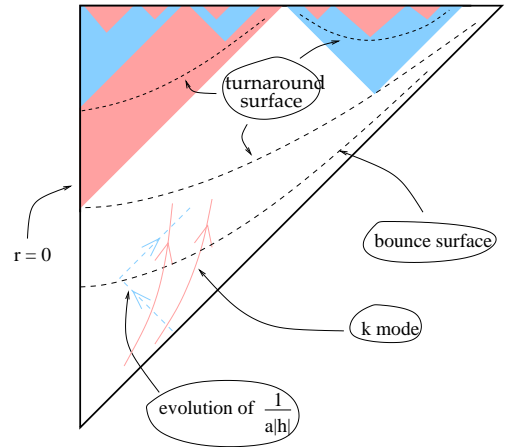


FIG. 4: The Penrose diagram of a cyclic multiverse. This is obtained by gluing the corresponding parts of diagrams of contracting universe, expanding universe and dS universe, where the dashed lines denoting the turnaround surface and the bounce surface have been signed respectively. Before each turnaround surface, the universe will experience the contraction, bounce, expansion successively and then enter into a period of dark energy domination. After this, the parent universe is separated into lots of parts independent of one another, each of which corresponds to a new universe and repeats above evolution. In principle, since the experience of each universe after the proliferation is generally different, they will not be expected to be synchronous in cycling. This means that generally when we are in a period of dark energy domination, it is possible that there are many other universes which are in the period of contraction or bounce or others.

The causal patch diagram of cyclic multiverse can be plotted in Fig.4, which is obtained by gluing the corresponding parts of diagrams of contracting phase, expanding phase and dS phase. This causal diagram is slightly similar to that of inflationary multiverse, e.g.[37]. The reason is that before the turnaround in each cycle the universe is in a dS phase. In some sense, such a period leads that the universe has an average positive energy density for each cycle. In inflationary multiverse, the new universe is generated either in nucleated bubble, e.g. [38],[39], or in the region that the inflaton field is in stochastic walking [40],[41], both are induced by quantum effects <sup>7</sup>. Here, however, the mechanism resulting in multiverse is the amplification of the amplitude of curvature perturbation cycle by cycle, which thus occurs in classical sense, though it initially originates from the quantum fluctuation of field.

We can have a concrete implement to this multiverse scenario, like in [5]. In this implement, the potential of

signals, which might be interesting for studying.

<sup>7</sup> However, in slow roll inflation, the multiverse can be also induced by the classical rolling of inflaton along a web of branches of its effective potential [42].

scalar field has a nearly flat region, the value of whose energy density equals to that of cosmological constant observed, and a minimum with negative energy density. When the field is in the nearly flat region of potential, we can have a period of dark energy, and then it rolls down towards its minimum with negative potential energy density, which leads to the collapse of universe, the contraction with  $w \simeq 0$  can be obtained by the oscillation of field around its minimum. In this case, it is obvious that the evolution that the period of dark energy domination precedes the contraction with  $w \simeq 0$  can be obtained. In general, the period of dark energy will help to dilute the matter and radiation in  $i$  cycle, which assures that in  $j$  cycle the energy density of matter and radiation from  $i$  cycle can not exceed that of field till the enough e-folding number is obtained. In addition, the anisotropy and some baneful leftovers, which are menaces for cycling, e.g. discussions in [3], can be also diluted during this period of dark energy. Thus it is required that this period should be enough long. i.e. the corresponding potential should be enough flat.

In different cycle of cyclic universe, the universe can be in different minima of a given potential in field space, or landscape<sup>8</sup> [5], in which these minima have negative potential energy density. Here, the generality is actually straight. Thus in principle it can be argued that in a cyclic multiverse scenario, generally each of multiverse will have different minima and thus evolutions. However, in this case, the requirement for an enough period of accelerated expansion in  $i$  cycle actually corresponds to a fine tuning for the initial condition of  $j$  cycle. Thus though in a cyclic universe driven by a given landscape, the number of universes having an enough long period of dark energy domination might be quite small, such universes can certainly exist, which is significant not only for assuring the continuance of cycle, but also for the emergence of observable universe in which we might live. In general, in a string landscape both positive and negative minima exist. However, unless all minima in the landscape are negative, the cycle of universe will not continue eternally, since if the universe enters into a positive minimum, it will stop cycling, and expand for ever, as illustrated earlier in [12].

In cyclic universe [2], the effect of the increase of metric perturbation on global universe has been discussed in [43]. However, in this model it requires that on super horizon scale the increasing mode of metric perturbation is inherited by the constant mode of curvature perturbation in leading order. Whether the corresponding inheriting can occur remains controversial [44],[45],[46],[47],[48],[49],[50],[51]. However, if this does not occur, the same mode is decayed during the expansion

of each cycle, thus even if it is increased during the contraction, the net result is still decayed in each cycle, since in each cycle the amount of the expansion of the universe is generally larger than that of the contraction. Here, however, the increasing mode of metric perturbation is inherited by the constant mode of curvature perturbation in  $k^2$  order, which certainly occurs for the bounce connecting the contracting and expanding phases, e.g. [29, 52] for theoretical and numerical studies. The metric perturbation after the bounce will be dominated by the same constant mode as that of curvature perturbation. Thus the net amplitude of metric perturbation is increased in each cycle. We have argued this effect will potentially lead to a cyclic multiverse [14]. In this paper, the problems baffling this multiverse scenario is discussed, and the possible solution is designed.

In a class of oscillating universe, in which the bounce is implemented by quintom matter [8],[20],[53], the cyclic multiverse scenario might be naturally applied, since a period of dark energy can be congenitally included in this model. This study can be in order. In [54], a scenario that many small contracting universes can be spawned at the end of each cycle has been proposed, which is implemented by appealing to the brane world cosmology and the phantom dark energy. In the design given here, these additional appeals are needless, the mechanism resulting in the multiverse is the natural increase of perturbation on super horizon scale. The role of a period of dark energy for cycling was also discussed in [55], in which the appearance of new universe is obtained by the wormholes leaded by the accretion of phantom dark energy [56].

In cyclic inflation [11],[12], the scale factor in consecutive cycles increases by a constant factor. This, after some cycles, can give a net exponential growth of the scale factor, and thus can imitate inflation. Whether there can be a scale invariant spectrum in this model remains an open issue. However, combining it with the result obtained here, we might argue that a multiverse will come into being after cyclic inflation, some of which might have a subsequent period of slow roll inflation and thus correspond to our observable universe.

In conclusion, in a cyclic universe, since the metric perturbation on super horizon scale is amplified cycle by cycle, after each cycle the universe will be inevitably separated into many parts independent of one another, each of which corresponds to a new universe and evolves up to succedent cycle, and then is separated again. This mechanism brings us a scenario of cyclic multiverse, in which the number of universes increases cycle by cycle. However, in general, the amplitude of perturbation modes in previous cycle can be preserved and amplified in current cycle, which is generally larger than that induced by the quantum fluctuation of background field in current cycle. This baffles the possibility that this scenario is regarded as the origin of observable universe. We, in this paper, have provided a viable design of a cyclic multiverse, in which the observable universe can emerge naturally. The significance of a long period of dark energy before the

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<sup>8</sup> In the low energy limit, the string landscape can be visualised as an effective potential in a given field space with multiple dimensions.

turnaround of each cycle for this implementing is shown. In this design, the causal diagram of cyclic multiverse likes that of inflationary multiverse. Thus the measure for the multiverse might be discussed similarly. Dark energy is an important issue of current cosmology, which is being intensively explored. In some sense, this work might provide an alternative motivation for the existence

of dark energy.

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